UDC 551.511.32: 551.509.324.2: 551.578.45

# NOTE ON ESTIMATION OF VERTICAL MOTION BY THE OMEGA EQUATION

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ABSTRACT

An interpretation of the $\omega$-equation shows that upward motion is closely related to the advection of vorticity in the upper troposphere and to warm advection in the lower troposphere. Examples are presented to illustrate both effects.

## 1. INTRODUCTION

The omega equation is one of the few that offer a satisfactory basis for an estimation of large-scale vertical motion. The principal terms in this equation relate the vertical motion to readily estimated advections. In this paper an interpretation of the omega equation is given which shows that the upward motion is closely connected to advection of vorticity in the upper troposphere and warm advection in the lower troposphere. A few examples from published papers (by other authors) are presented in a modified form to illustrate both effects.

## 2. THE OMEGA EQUATION

The geostrophic and adiabatic assumptions lead to the omega equation, which may be rewritten in the form derived from equation (9.14) of Thompson (1961):

$$-\left(-\sigma\nabla^2\omega+\frac{f^2}{g}\frac{\partial^2\omega}{\partial p^2}\right)=\frac{f}{g}\frac{\partial}{\partial p}(-\mathbf{V}\cdot\nabla\eta)+\frac{R}{pg}\left(\frac{p}{p_0}\right)^k\nabla^2(-\mathbf{V}\cdot\nabla\theta), \tag{1}$$

where $\nabla^2$ is the horizontal Laplacian and the other symbols have the conventional meaning.

A simplification of equation (1) can be made by observing that the sign of the left-hand side coincides with the sign of the three-dimensional Laplacian, $\nabla_3^2$. Also, since we are interested only in qualitative estimates, the value of various constants becomes irrelevant. Therefore, the constant factors can be omitted. Then equation (1) may be approximated by the proportionality

$$-\nabla_3^2\omega\propto\frac{\partial}{\partial p}(-\mathbf{V}\cdot\nabla\eta)+\nabla^2(-\mathbf{V}\cdot\nabla\theta). \tag{2}$$

Vertical motion in weather systems varies around the zero value. Therefore, one may use a further proportionality

$$-\nabla_3^2\omega\propto\omega.$$

It may be kept in mind that this last proportionality could be applied also in the last term in equation (2).

The values of $\omega$ and $\mathbf{V}\cdot\nabla\theta$ increase proportionally to each other from usually small values at the earth's surface. Therefore, the estimates of $\omega$ at levels above the surface depend greatly on accumulated values of $-\mathbf{V}\cdot\nabla\theta$ from the surface up to the level. Consequently, for the purpose of estimating $\omega$, the thermal advection should be considered for thick layers of lower troposphere and not only at isolated levels.

The term $\frac{\partial}{\partial p}(-\mathbf{V}\cdot\nabla\eta)$ represents the change with pressure of the vorticity advection. The advection of vorticity usually increases with height without change of sign, much in the way wind increases with height. Therefore in layers under the region of maximum wind, a good proportionality is given by

$$\frac{\partial}{\partial p}(-\mathbf{V}\cdot\nabla\eta)\propto\mathbf{V}\cdot\nabla\eta.$$

Hence, in the troposphere under the jet-stream level, it follows that

$$\omega\propto\mathbf{V}\cdot\nabla\eta+\nabla^2(-\mathbf{V}\cdot\nabla\theta).$$

Transforming to the geometrical vertical motion, $w=\frac{dz}{dt}\approx-\frac{\omega}{g\rho}$, the relation becomes

$$w\propto-\mathbf{V}\cdot\nabla\eta-\nabla^2(-\mathbf{V}\cdot\nabla\theta). \tag{3}$$

From the foregoing, it follows that the upward motion in a midtropospheric level is primarily associated with:

1) positive vorticity advection above that level, and
2) maximum warm advection below that level.

The reverse is true for downward motion ($w<0$). (In point 2, in fact, the Laplacian of advection should be considered, but a simplified expression, as given, may be sufficient in most cases.) Warm advection may be estimated also from thickness advection since $\theta$ is proportional to thickness.

For manual analysis of weather charts, hardly anything will yield more accurate estimates than recognition of the above points. If the two processes counteract, an estimate of $w$ cannot be made. In cases where both processes show the same sign in a region, a qualitative estimate of vertical motion can be made. Strong upward motion should cause heavy precipitation of the continuous type.

The two "rules" for vertical motion of $w>0$ coincide with the two (out of four) main effects causing cyclogenesis, see equation (16.2.11) in Petterssen (1956).

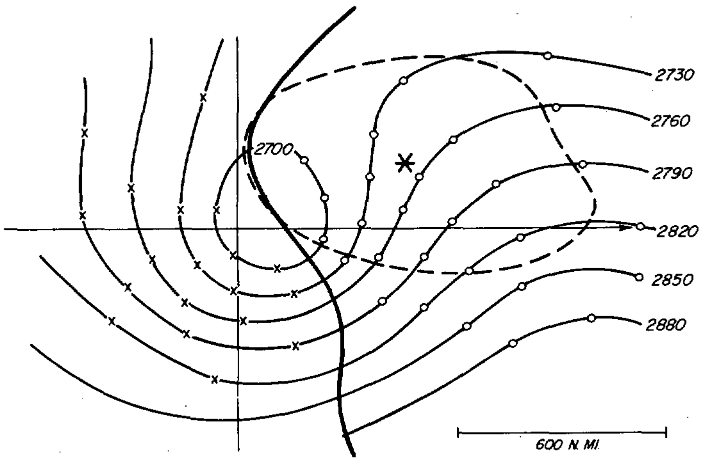


FIGURE 1.—Advection of temperature in the composite case given by Goree and Younkin (1966). Each solid contour shows the average geopotential $(z_{500}+z_{1000})/2$. Warm advection is in the region of small circles; cold advection is behind the heavy line. Following Goree and Younkin (1966), the broken line is the 0 percent probability of occurrence of heavy snowfall, within which the probability ranges to a maximum of over 40 percent at the * symbol. The very heavy line is the isoline of zero thermal advection.

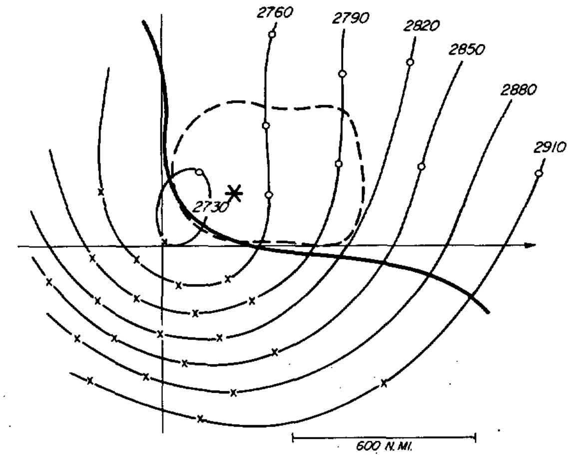


FIGURE 2.—Symbols as in figure 1; this case valid for the "digging" type in figures 1 and 2 of Younkin (1968); the broken line is the 10 percent probability of heavy snowfall.

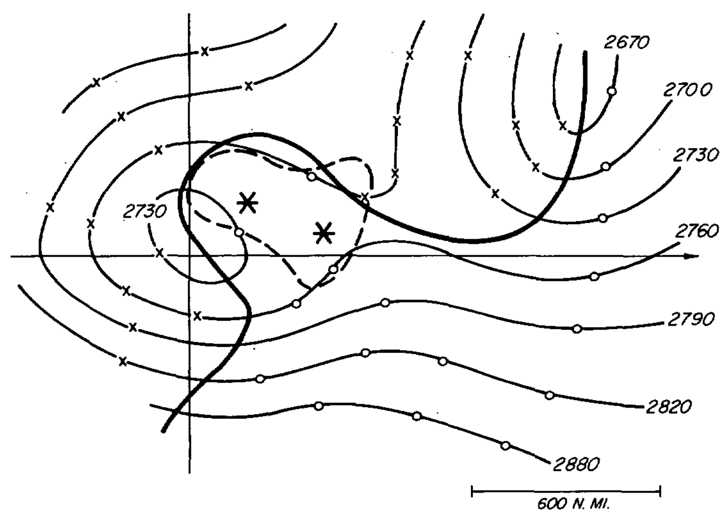


FIGURE 3.—Symbols as in figure 1; this case valid for the "coming out" type in figures 3 and 4 of Younkin (1968); the broken line is the 10 percent probability of heavy snowfall.

The fact that the adiabatic equation is used in equation (1) should not change the above conclusions about vertical motion. In case of descending motion, the adiabatic assumption holds fairly well. In case of upward motion with condensation of water vapor, the addition of latent heat may be significant. However, in this case the upward motion should be accelerated by heating. Thus the rules about vertical motion can be supported in case of release of condensation heat.

A similar analysis of the omega equation was made by Wiin-Nielsen (1959). He reduced the estimate of vertical motion to the estimate of the advection of temperature in the isolines of vorticity, using data from one level only.

## 3. EXAMPLES

Appearance of intensive ascending motion (large-scale) is certain in cases of heavy snowfall. (Heavy snowfall is not likely connected with local thunderstorms.) The examples published by Goree and Younkin (1966) and Younkin (1968) may be considered typical for ascending motion. On the basis of charts published by these authors, figures 1 to 3 have been constructed.

As in the source papers (Goree and Younkin, 1966; Younkin, 1968), the center of the coordinate system in figures 1 to 3 is situated in the point of maximum vorticity at 500 mb. The abscissa (mainly downstream) points in the direction of movement of the vorticity center, as concluded from the series of charts. It may be assumed that the motion of the vorticity centers is primarily due to the horizontal advection. Therefore, the maximum of vorticity lies in the vicinity of the abscissa, downstream from the vorticity maximum.

The thermal advection in the lower troposphere is evaluated graphically as an advection of the 500- to 1000-mb thickness by an average geostrophic flow in that layer. The average flow is shown in figures 1 to 3 by the mean contours $\bar{z}$, averaged by

$$\bar{z}=\frac{z_{1000}+z_{500}}{2}.$$

The 500- to 1000-mb thickness and the heights at 1000 and 500 mb were taken from the original figures referred to in the captions. In figures 1 to 3, the warm advection is proportional to the number of small circles per unit area. Cold advection is similarly represented by crosses.

Crosses and circles are located at intersections of contours of $\bar{z}$ and thickness.

In figures 1 to 3, as well as in the original figures by Younkin, and Goree and Younkin, it is remarkable that the maximum probability of heavy snowfall appears to the left of the direction in which the vorticity maximum has moved. This fact is easier to understand when the distribution of thickness advection is considered. In the given examples, either the thickness advection is most pronounced to the left of the abscissa or the zero advection line shows a curvature which may cause significant values of the Laplacian of the advection, see proportionality (3).

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*[Received February 14, 1969; revised June 2, 1969]*